Kuwait University, Faculty of Science,
Dept. of Mathematics & Computer Science
Calculus I (Math 101) Second Mid-Term Test

July 19, 2007 | Duration 90 minutes

Calculators and mobile phones are NOT allowed

Answer the following questions:

1. [2 pts.] Find the derivative of $f(x) = \sqrt{2x+1}\ \sec^3(5 - x^2)$.

2. [3 pts.] Show that the curves $x^2 + y^2 = r^2$ and $y = mx$ are orthogonal (meet at right angles) for all constants $m \neq 0, r \neq 0$.

3. [3 pts.] Use differentials to approximate $7 + (1.02)^4$.

4. [4 pts.] A point P moves on the curve $y = x + 5$ such that $\frac{dx}{dt} = 3$ units/sec. Find the rate of change of the distance between P and the point $Q(2,0)$ when P is at $(-5,0)$.

5. [4 pts.]

   (a) State Rolle's theorem.

   (b) Let $f(x) = \begin{cases} x & if\ 0 \le x \le 1 \\ 2 - x & if\ 1 < x \le 2 \end{cases}$.

   i. Show that $f(0) = f(2)$.

   ii. Show that $f'(c) \neq 0$ for all $c \in (0,2)$.

   iii. Does this contradict Rolle's theorem? Explain.

6. [9 pts.] Let $f(x) = \frac{x^2+1}{x}$.

   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.

   (b) Find the intervals on which f is increasing or decreasing and find the local extrema, if any.

   (c) Find the intervals on which the graph of f is concave upward or concave downward and find the points of inflection, if any.

   (d) Discuss the symmetry of the graph.

   (e) Sketch the graph of f.

1. $f'(x) = \sec^3(5-x^2)\,\frac{1}{\sqrt{2x+1}} + \sqrt{2x+1}\cdot 3\sec^3(5-x^2)\tan(5-x^2)(-$

2. $x^2+y^2=r^2 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow \frac{dy}{dx}\Big|_{y=mx} = -\frac{x}{mx} = -\frac{1}{m}$.
$y = mx \Rightarrow \frac{dy}{dx} = m \Rightarrow$ Two Curve meet at right angle.

3. $f(x) = 7+x^4$, $f'(x) = 4x^3$. $x=1$ and $\Delta x = 0.02$.
$f(x+\Delta x) \cong f(x) + f'(x)\Delta x \Rightarrow f(1.02) \cong f(1) + f'(1)(0.0$
$7+(1.02)^4 \cong 8 + (4)(0.02) = 8.08$.

4. $\overline{PQ}^2 = \ell^2 = (x-2)^2 + (y-0)^2 = (x-2)^2 + (x+5)^2 = 2x^2+6x+29$
$\frac{d}{dt} \Rightarrow \quad 2\ell\frac{d\ell}{dt} = (4x+6)\frac{dx}{dt}$
when $x=-5$, $\ell=7 \Rightarrow (2)(7)\frac{d\ell}{dt} = (-14)(3)$
$\Rightarrow \frac{d\ell}{dt} = -3$ units/sec.

5. b) i. $f(0) = 0 = f(2)$
ii. $f'(x) = \begin{cases} 1 & \text{if } 0<x<1 \\ -1 & \text{if } 1<x<2 \end{cases} \Rightarrow f'(c) \neq 0 \ \forall c \in (0,2)$
iii. No, as $f(x)$ is not differentiable at $x=1$.

6. a) $\lim_{x\to 0^{\mp}} f(x) = \mp\infty \Rightarrow \boxed{x=0}$ is a V.A.
(2) $\lim_{x\to\infty(-\infty)} f(x) = \infty(-\infty) \Rightarrow$ No H.A.s.

b) (2) $f'(x) = \frac{x^2-1}{x^2}$
$f'(x)=0 \Rightarrow x=\pm1$
$f'(x)$ DNE $\Rightarrow x=0$

	$-\infty$ -1	0	1	∞
Intervals	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Sign of f'	+	−	−	+
Conclusion	↗	↘	↘	↗

Local max: $f(-1) = -2$
Local min: $f(1) = 2$

c) (2) $f''(x) = \frac{2}{x^3}$.
$f''(x)$ DNE $\Rightarrow x=0$
No P.I.

	$-\infty$ 0	∞
Intervals	$(-\infty,0)$	$(0,\infty)$
Sign of f''	−	+
Concavity	∩	∪

d) (1) $f(-x) = -\frac{x^2+1}{x} = -f(x)$
The graph is symmetric w.r.t to the origin.

e) (2)